Lecture 8 - Oct. 1

Math Review, Lab1 Solution, Lab2

Algebraic Properties of Relational Ops
Lab2: Celebrity_0
Functional Properties

Announcements/Reminders

- Lab2 due this Friday
- Guide for Programming Test 1 released

Relational Operations: Overriding

$\{(x,y) \mid P_1(x,y) \lor P_2(x,y)\}$
$= \{(x,y) \mid P_1(x,y)\} \cup \{(x,y) \mid P_2(x,y)\}$

r = {~~(a, 1)~~, (b, 2), ~~(c, 3)~~, ~~(a, 4)~~, (b, 5), ~~(c, 6)~~, (d, 1), (e, 2), (f, 3)}

Example: Calculate r overridden with {(a, 3), (c, 4)}

Hint: Decompose results to those **in** t's domain and those **not in** t's domain.

$$r \Leftarrow t = \{(d, r') \mid (d, r') \in t \lor ((d, r') \in r \land d \notin dom(t))\}$$

relation relation

rel r is overridden by rel t

$$= \{(d, r') \mid (d, r') \in t\} \cup \{(d, r') \mid (d, r') \in r \land d \notin dom(t)\}$$

r subtraction

$$= t \cup (dom(t) \lhd\!\!\!- \; r)$$

$\{(b,2), (b,5), (d,1), (e,2), (f,3)\}$

$$r \Leftarrow \underbrace{\{(a,3), (c,4)\}}_{t} = \underbrace{\{(a,3), (c,4)\}}_{t} \cup \underbrace{\boxed{(\{a,c\} \lhd\!\!\!- \; r)}}_{(dom\ t) \lhd\!\!\!- \; r} = \boxed{}$$

$r = \{\cancel{(a, 1)}, (b, 2), \cancel{(c, 3)}, \cancel{(a, 4)}, (b, 5), \cancel{(c, 6)}, (d, 1), (e, 2), (f, 3)\}$

$(a, 3)$, $(c, 4)$

Example: Calculate r overridden with $\{(a, 3), (c, 4)\}$

$b \in$ Account

$\Leftrightarrow$

t

transfer.

$\boxed{\text{changes}} = \{(a1, b(a1) - amt), (a2, b(a2) + amt)\}$

basically r ,

excep all pairs with first elements in the domain of t should agree with t.

THINK: $\boxed{\text{transfer event from Cab1}}$

Exercises: Algebraic Properties of Relational Operations

$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$

$r[s]$ $\qquad s = \{a, b\}$

Define the image of set s on r in terms of other relational operations.

Hint: What range of value should be included?

$\boxed{r[s]} = \mathrm{ran}(\underbrace{s \lhd r}_{\text{gives a relation}})$

$\subseteq \mathrm{ran}(r)$

✓

Define r overridden with set t in terms of other relational operations.

Hint: To be in t's domain or not to be in t's domain?

$r \oplus t = t \cup (\mathrm{dom}(t) \lhd r)$

$$r \in \underbrace{\boxed{S}}^{\{a,b,c,d\}} \longleftrightarrow \boxed{T}\ \{1,2,3,4\}$$

$$\underline{\underline{s}} \subseteq S \qquad t \subseteq T$$

Problem: reconstruct relation r in terms of restriction & subtraction.

$$r \stackrel{?}{=} (s \lhd r) \cup (s \mathbin{⩤} r)$$

$$r \stackrel{?}{=} (r \rhd t) \cup (r \mathbin{⩥} t)$$

<u>Examples</u>

$$r = \{(a,1),(b,2),(c,3)\}$$

$$s = \{b,d\}$$

$$t = \{1,4\}$$

Lab2: Relational Operators

$k^{\sim} = \{(Mark, Alan), (Tom, Alan), (Tom, Mark)\}$

Among persons, there is a special one: the celebrity. We define the *knows* relation which captures how the celebrity and other persons should be treated differently: $c \in P$

$id = \{(Alan, Alan), (Mark, Mark), (Tom, Tom)\}$

1. No person (including the celebrity) knows himself or herself.
2. The celebrity knows no one.
3. Everyone know the celebrity.

1. celebrity is known by everyone, except themselves

$k \in P \leftrightarrow P$

$k \in \mathbb{P}(P \times P)$

e.g. $P = \{Alan, Mark, Tom\}$

$k = \{(Alan, Mark), (Alan, Tom), (Mark, Tom)\}$

$\subseteq P \times P$

CONTEXT Celebrity_c0

CONSTANTS

k knows relation

c celebrity

P Set person

AXIOMS

axm1: $\boxed{P \subseteq \mathbb{N}}$ model the set of persons via unique ids.

axm2: $\boxed{c \in P}$ typing constraint

axm3: $k \in (P \setminus \{c\}) \leftrightarrow P$ $c \notin dom(k)$

axm4: $k^{-1}[\{c\}] = P \setminus \{c\}$

axm5: $\boxed{k \cap id = \varnothing}$ identity relation

END

axm5

$\forall (p1, p2) \cdot (p1, p2) \in k \Rightarrow p1 \neq p2$

MACHINE Celebrity_0

SEES Celebrity_c0

VARIABLES

r

INVARIANTS

inv1: $r \in P$

EVENTS

Initialisation

begin

act1: $r :\in P$

end

Event celebrity $\langle$ordinary$\rangle$ $\widehat{=}$

begin

act1: $r := c$

end

END

axm 4 : $\underbrace{k^{-1}[\{c\}]}_{\text{celebrity is known by}} = \boxed{P \setminus \{c\}}$

tom

every person except the celebrity themselves

$k^{\sim} = \{(\text{Mark}, \text{Alan}),$
$(\text{Tom}, \text{Alan}),$
$(\text{Tom}, \text{Mark})\}$

$\boxed{k^{\sim}[\{\text{Tom}\}]} = \boxed{\{\text{Alan}, \text{Mark}\}}$

celebrity Tom is known by

everyone except Tom

axm 3

$k \in (P \setminus \{c\}) \leftrightarrow P$

$\equiv$

$k \in P \leftrightarrow P$

$c \notin \mathrm{dom}(k)$

Functional Property

$r \in S \leftrightarrow T$

$\text{isFunctional}(r) \Leftrightarrow$

e.g. $\{(\underbrace{a}_{s}, \underbrace{1}_{t1}), (b, 2), (\underbrace{a}_{s}, \underbrace{3}_{t2})\}$

$\forall s, t1, t2 \bullet$

$(\boxed{s \in S} \land \boxed{t1 \in T \land t2 \in T})$

dom val   ran val.

$\Rightarrow$

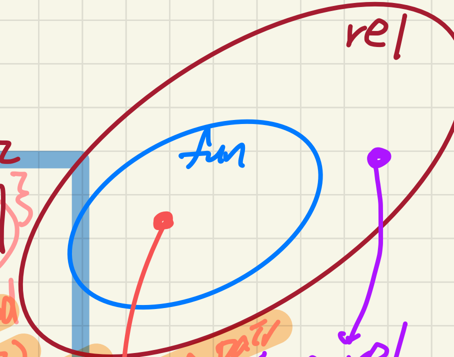
two pairs sharing the same 1st value

$(\boxed{(s, t1) \in r \land (s, t2) \in r} \Rightarrow \boxed{t1 = t2})$

the same dom value cannot map to distinct values in the range.

(s, t1) and (s, t2) refer to the same pair

2nd values are the same



some rel not qualified to be a fun.

a rel and a function at the same time.

Q: Smallest relation satisfying the functional property. $\emptyset$

Q: How to **prove** or **disprove** that a relation r is a function.

Q: Rewrite the functional property using contrapositive.

$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$

$r = \{ (a, \boxed{1}), (b, 2), (c, \boxed{1}) \}$

1. Exercise: check this against the func. property

→ satisfied

2. Injective function